Exercises

Vectors and Matrices

Exercise 1. Consider the vectors $u = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, and $w = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$.

Compute:

a) $3u + 2v - w$

b) $w - (\boldsymbol{e}_1 - \boldsymbol{e}_2) + \boldsymbol{e}_3$

c) $\frac{1}{2}(u - \mathbf{1}) + 4(v - w)$

Note: $\boldsymbol{e}_i$ is the i-th unit vector and $\mathbf{1}$ is the all-one-vector.

Exercise 2. Given the vectors $\mathbf{g} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $\mathbf{h} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ and the matrices

$$A = \begin{bmatrix} 5 & -1 & 2 \\ -8 & 3 & 7 \end{bmatrix}, \; B = \begin{bmatrix} 4 & -5 & -2 \\ -10 & -1 & -3 \end{bmatrix}, \; C = \begin{bmatrix} 6 & 8 \\ 0 & 2 \end{bmatrix}$$

compute the following expressions (if possible):

a) $A + B$

b) $AB, \quad AB^\top, \quad BA^\top$

c) $A\mathbf{1}, \quad \mathbf{e}_2^\top A$
($\mathbf{e}_i$ is the i-th unit-vector and $\mathbf{1}$ is the all-one vector of appropriate size)

d) $\mathbf{g}^\top A^\top, \quad \mathbf{g}^\top \mathbf{h}, \quad \mathbf{g}\mathbf{h}^\top$

Exercise 3. A vector v is called **normalized** if $\|v\| = 1$. Two vectors v, w are called **orthogonal** if $\langle v, w \rangle = 0$. Two vectors v, w are called **orthonormal** if they are normalized and orthogonal.

a) For which $a \in \mathbb{R}$ is $(a, -3a)^\top$ a normalized vector?

b) Find all vectors that are orthogonal to $u^\top = (5, -1)$.

c) Normalize the vectors $v = (-2, 4, -5, 2)^\top$, $w = (2, -1, 3)^\top$.

d) Find all vectors that are orthonormal wrt. $(2, -3)^\top$.

Exercise 4. Assume some company produces three intermediate products I_1, I_2, I_3 from the four different resources R_1, R_2, R_3, R_4 and finally three end products P_1, P_2, P_3 from the intermediate products.

Resource	Used resources per intermediate product I_1	I_2	I_3
R_1	0	3	1
R_2	1	2	2
R_3	3	1	1
R_4	2	0	2

I-Prod.	Used intermediate products per end product P_j: P_1	P_2	P_3
I_1	1	2	1
I_2	0	3	2
I_3	4	1	2

1. Draw a scheme that visualizes this 2-step process.

2. Assume the company wants to produce 50 units of P_1, 100 units of P_2, and 200 units of P_3. How many intermediate products must be produced and how many resources must be bought?